National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"

Applied Mathematics
Department

Game Theory

Working Programme of an Academic Discipline (Syllabus)

Academic Discipline Description

Level of Higher Education	***Second (Master's)***
Field of Knowledge	*F Information Technology*
Major	*F1 Applied Mathematics*
Educational Programme	*Machine Learning and Mathematical Modelling*
Discipline Status	*Elective*
Form of Study	*Full-time (day)*
Year, Semester	*1 year, Spring semester*
Total workload	*5 ECTS credits / 150 h: (30 h – lectures, 16 h — sections, 104 h – self-study)*
Semester Assessment / Assessment Measures	*Exam, Calculation and Graphic Assignment (CGA), Midterm Test*
Class Schedule	*http://schedule.kpi.ua* *Lectures: once a week* *Sections: once every two weeks*
Language of Instruction	*English*
Course Leader / Instructor Information	Lectures and Sections: Danylo Tavrov, Assoc. Prof., PhD e-mail: tavrov.danylo@lll.kpi.ua
Course Location	

Course Syllabus

1. Course Description, Purpose, Subject Matter, and Learning Outcomes

The purpose of this course is for students to master the conceptual foundations, mathematical apparatus, and methods of game theory required to analyse strategic interactions between rational agents; to develop skills in constructing and solving game-theoretic models across various applied domains; and to foster strategic thinking and the ability to predict outcomes in complex, competitive scenarios.

Subject Matter of the Course:

- Static Games of Complete Information: strategic form of a game, best response, Nash equilibrium in pure and mixed strategies, strategy dominance;
- Dynamic Games of Complete Information: extensive (positional) form of a game, backward induction, subgame perfect Nash equilibrium, repeated games;
- Games of Incomplete Information: static and dynamic interactions under uncertainty regarding player types, Bayesian Nash equilibrium, and its refinements.

After completing the course, students must demonstrate the following competencies and programme learning outcomes in accordance with the educational programme:

General competencies

ЗК01. Ability for self-learning, searching, processing, and intellectually analysing information from various sources, as well as ability to identify, pose, and solve problems.

Professional competencies

ФК01. Ability to formalise and solve complex problems that require updating and integrating knowledge, often in conditions of incomplete, inaccurate, or insufficient information and conflicting requirements.

Programme learning outcomes

ПРН01. To utilise and adapt mathematical theories and models to provide a theoretical basis for solving scientific and practical problems.

ПРН02. To apply existing mathematical tools, develop new models, methods, and algorithms to solve a wide range of practical problems.

2. Course Prerequisites and Postrequisites

A foundational knowledge of probability theory, mathematical analysis (calculus), discrete mathematics, and proficiency in Python programming will be beneficial for the successful completion of this course.

3. Course Content

SECTION 1. STATIC GAMES OF COMPLETE INFORMATION

Topic 1.1. Strategic Form of a Game and Pure Strategy Nash Equilibrium

Topic 1.2. Mixed Strategies and Mixed Strategy Nash Equilibrium

Topic 1.3. Strategy Dominance and Iterated Elimination of Strictly Dominated Strategies

SECTION 2. DYNAMIC GAMES OF COMPLETE INFORMATION

Topic 2.1. Extensive Form of a Game: Decision Trees and Information Sets

Topic 2.2. Backward Induction and Subgame Perfect Nash Equilibrium

Topic 2.3. Models of Multi-Stage Interaction and Alternating-Move Games

Topic 2.4. Repeated Games and the Folk Theorem

SECTION 3. GAMES OF INCOMPLETE AND IMPERFECT INFORMATION

Topic 3.1. Static Games of Incomplete Information: Bayesian Nash Equilibrium

Topic 3.2. Dynamic Games of Incomplete Information: Perfect Bayesian Equilibrium

Topic 3.3. Signalling Games, Separating and Pooling Equilibria Conditions

Topic 3.4. Applications in Economic and Social Models (Auctions, Bargaining)

4. Learning Materials and Resources

Primary Reading

1. Bonanno G. Game Theory. 2024. 634 p. URL: https://faculty.econ.ucdavis.edu/faculty/bonanno/PDF/GT_book.pdf (access date: 25.12.2024)

2. Karlin A. R., Peres Y. Game Theory, Alive. 2016. 393 p. URL: homes.cs.washington.edu/~karlin/GameTheoryBook.pdf

Auxiliary Reading

3. Osborne M. J., Rubinstein A. A Course in Game Theory. MIT Press, 1994. 352 p. URL: https://sites.math.rutgers.edu/~zeilberg/EM20/OsborneRubinsteinMasterpiece.pdf (access date: 25.12.2024).

Course Content

5. Teaching Methodology

Instruction is delivered using a student-centred approach built on active interaction between the instructor and students to facilitate material comprehension and practical skill development. Lectures and sections employ explanatory-illustrative methods alongside problem-based learning techniques.

In the case of remote instruction, classes are conducted via the Zoom video conferencing platform.

Lectures

#	Lecture Title and Main Topics
1	Introduction to Game Theory Subject matter of game theory and its place among scientific disciplines; concepts of rationality and common knowledge; classification of games.
2	Strategic Form of a Game and Nash Equilibrium Formal definition of a game in strategic form; sets of players, strategies, and payoffs; the concept of stable interaction; definition of pure strategy Nash equilibrium.
3	Best Response Function Best response operator; relationship between best response and Nash equilibrium; graphical method for finding equilibrium in games with continuous strategy sets.
4	Mixed Strategies Expanding the strategy space; concept of randomization; expected payoff; existence of Nash equilibrium in mixed strategies (Nash's Theorem).
5	Dominance and Minimax Strictly and weakly dominated strategies; iterated elimination of dominated strategies; zero-sum games and von Neumann's minimax theorem.
6	Extensive Form of a Game Game tree; decision nodes and terminal nodes; information sets; defining a strategy as a complete plan of action; backward induction; subgame perfect Nash equilibrium.
7	Extensive Form Games with Simultaneous Moves Extensive form games with simultaneous moves; committee decision-making.
8	Extensive Form Games with Exogenous Uncertainty Extensive form games with exogenous uncertainty; limitations of subgame perfect Nash equilibrium.
9	Repeated Games (Part 1) Structure of repeated interaction; discount factor and time preference; Nash Folk Theorem.
10	Repeated Games (Part 2) Subgame perfect Nash equilibria for infinitely repeated games; Perfect Folk Theorem; application to efficiency wage setting.
11	Strategic Games with Incomplete Information Definition of a Bayesian game; simple applications; impact of information on equilibria; mixed strategies as the limit of pure Bayesian strategies.
12	Auction Theory Classification of auctions (English, Dutch, first-price, and second-price); Revenue Equivalence Theorem; auctions with budget constraints.
13	Mechanism Design (Part 1) Concept of direct mechanisms; Revelation Principle; optimal single-agent mechanisms.
14	Mechanism Design (Part 2) Optimal auctions; VCG mechanisms; public good provision; optimal double auction.
15	Perfect Bayesian Equilibrium (PBE) Sequential rationality requirement; belief systems and updating via Bayes' rule; formal definition of PBE.

Sections

#	Section Title and Main Topics
1	Examples of Strategic Games Voting models, war of attrition, auctions.
2	Examples and Analysis of Games in Mixed Strategies Solving 2 x 2 games in mixed strategies; games with no pure strategy equilibrium (Matching Pennies); interpreting mixed strategies as behavioural frequencies.
3	Examples of Extensive Form Games Parlor games; simple bargaining models; vote buying; races.
4	Midterm
5	Bargaining Models Two-stage resource allocation model; Rubinstein's infinite-horizon bargaining; impact of the discount factor on player bargaining power.
6	Evolutionary Equilibrium Monomorphic equilibria in pure strategies; polymorphic strategies in mixed strategies; asymmetric equilibria.
7	Examples of Strategic Games with Incomplete Information Public good provision; double auction; war of attrition.
8	Types of Equilibria in Signalling Games Separating equilibrium; pooling equilibrium; semi-separating equilibria; Cho–Kreps Intuitive Criterion for equilibrium refinement.

6. Independent Student Work

Students are expected to prepare for lectures and sections in advance. Prior to each lecture or section, students must review previously covered theoretical material and prepare questions regarding the upcoming topic for lecture discussion. In addition, students are required to complete a Calculation and Graphic Assignment (CGA) during the semester.

#	Type of Work	Hours of Work
1.	Completion of CGA	30
2.	Preparation to lectures	20
3.	Preparation to sections	20
4.	Preparation to midterm	4
5.	Preparation to final exam	30
	Total	104

Course Policies and Assessment

7. Course Policy

Class Attendance

In accordance with Order No. 1-273 dated 14 September 2020, evaluating or grading a student's physical presence or absence during scheduled classes is prohibited. Course material missed for any reason must be mastered independently. Nevertheless, students are strongly encouraged to attend all classes, as lectures cover core theoretical concepts and foster skills essential for practical assignments. The grading system focuses on awarding points for student participation as well as completing assignments that build practical skills and competence.

Rules for the Calculation and Graphic Assignment

Students complete the CGA throughout the semester by solving problems and programming game-theoretic models. The assignment must be completed individually, using Python (or any other programming language). Results must be submitted electronically to the instructor. The use of reference materials is permitted while working on the CGA.

Rules for the Midterm Test

The Midterm Test is administered as follows:

– in-person instruction: conducted during a designated section slot within a fixed timeframe, submitted in physical paper format;

– remote instruction: completed synchronously during a designated section slot within a fixed timeframe, submitted electronically to the instructor.

The use of reference materials during the Midterm Test is permitted; however, discussing problems with peers or seeking assistance from third parties is strictly prohibited.

A student may retake a missed Midterm Test only if the absence was due to valid, documented reasons. Retaking the test is subject to prior arrangement with the instructor.

Examination Rules

The examination is conducted in written form within a specified timeframe. It is submitted as a physical paper document during in-person delivery, or submitted electronically to the instructor in the case of remote instruction. Students must complete the examination independently. The use of reference materials is permitted; however, discussing problems with peers or seeking advice from third parties is strictly prohibited. The written section may be followed by an oral discussion covering the contents of the examination paper.

Missed Assessment Measures

A missed examination is not graded; in such cases, the student is marked as “Absent” in the official grade sheet and must take the exam during the supplementary examination period.

Academic Integrity

Policies and principles of academic integrity are defined in Section 3 of the Code of Honour of National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”. For details, visit: https://kpi.ua/code.

Codes of Ethical Conduct

Standards of ethical student conduct are defined in Section 2 of the Code of Honour of National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”. For details, visit: https://kpi.ua/code.

Appeals Procedure for Assessment Results

Students may raise any query regarding the procedure and/or evaluation of assessment measures and expect it to be handled in accordance with established procedures. Students have the right to appeal assessment results provided they present a reasoned justification explaining which specific rubric criterion or evaluation feedback they contest.

8. Types of Assessment and Grading Criteria

Continuous Assessment: CGA, Midterm Test.

Calendar Control: Conducted twice per semester (during Weeks 7 and 13) to monitor student compliance with syllabus requirements. A student receives a “Pass” if their current cumulative score is at least 50% of the maximum possible points achievable by that point in the semester.

Semester Assessment: Final Examination.

Prerequisites for Exam Admission: Satisfactory completion of all stages of the CGA and a cumulative semester score of at least 25 points.

The student's overall grade for the course module consists of:

1) points awarded for the CGA;
2) points for the Midterm Test;
3) points awarded for the Final Examination.

RATING POINT SYSTEM

1. Calculation and Graphic Assignment (CGA)

Assessment is based on the performance in the CGA, which is divided into three stages corresponding to the core modules of the course. The maximum cumulative score for all stages combined is **36 points**.

Each stage consists of 3 theoretical problems and 1 programming task. Each problem/task is worth **3 points**.

Evaluation Criteria for Each Problem/Task:

3 points – the problem is fully completed:

For theoretical problems: A correct analytical solution is obtained with logical justification provided;

For programming tasks: The code is functional, the algorithm accurately reflects the theoretical model, and simulation results are correctly interpreted;

2 points – The problem is completed with minor deficiencies;

For theoretical problems: The solution approach is correct, but contains non-critical arithmetic errors;

For programming tasks: The overall algorithm is correct, but the code requires adjustments for edge cases or visual output/plots are missing.

0 points – The problem is incomplete, or the solution contains fundamental errors demonstrating a lack of conceptual understanding.

2. Midterm Test

Students take the Midterm Test during a scheduled section. The test consists of four problems, 2 of which are evaluated at 3 points, and 2 of which are evaluated at 4 points. The maximum cumulative score for the Midterm Test is **14 points**.

Evaluation Criteria for Each Problem:

3 (4) points – The problem is solved correctly;

2 (3) points – The problem is solved mostly correctly, with minor deficiencies or non-critical arithmetic errors;

0 points – The solution is missing.

3. Final Examination Points

The examination paper consists of 1 theoretical question (proving a specific mathematical proposition/theorem) worth 20 points, and 3 problems worth 10 points each. The maximum examination score is 50 points.

Evaluation Criteria for the Theoretical Question:

20 points – The answer is complete and correct;

18-19 points – The answer is mostly correct, with minor flaws or slight terminological inaccuracies;

15-17 points – The answer contains notable conceptual errors;

12-14 points – The answer contains major errors;

0 points – No answer is provided.

Evaluation Criteria for Each Individual Problem:

10 points – The problem is solved correctly;

8-9 points – The problem is solved mostly correctly, with minor flaws or non-critical arithmetic errors;

6-7 points – The problem is partially solved or contains noticeable errors;
0 points – No solution is provided.

Maximum points for the Final Examination:

20 points × 1 theoretical question + 10 points × 3 problems = **50 points**

Calculation of the Overall Rating Scale (R):

The sum of weighting points for continuous assessment during the semester is:

$$R_C = 36 + 14 = 50 \text{ points.}$$

Out of this total, a student accumulates a specific number of semester points, denoted as r_C.

The examination component (R_E) accounts for 50 points, representing 50% of R.

A student earns a specific number of examination points, denoted as r_E, based on their exam performance.

The total rating scale for the course is:

$$R = R_C + R_E = 100 \text{ points}.$$

A mandatory prerequisite for exam admission is the completion of all stages of the CGA, along with accumulating a minimum of r_C = 25 points.

If a student fails to fulfil the exam admission criteria, they receive an entry of "Not Admitted" on the primary grade sheet. The student must then complete the outstanding requirements (continuing work on the CGA stages until sufficient points are achieved). Upon gaining admission, the student takes the examination (during the first and/or second resit session) according to the aforementioned rules. Their final course score (R_D) is calculated as: $R_D = r_C + r_E$.

In the case of remote summative assessment, the final mark is calculated according to the following formula, provided the student has completed all CGA stages and earned at least r_C = 25 points. The conversion of the initial semester rating score into the 100-point scale is carried out according to the formula

$$R_D = 60 + \frac{40(r_c - 25)}{25}$$

Retaking or re-assessing the final 100-point score calculated by the formula is not permitted. If the prerequisites for exam admission are not met by the end of the semester, the student receives a mark of "Not Admitted" on the primary grade sheet. The student must subsequently fulfil the admission requirements prior to the first resit session, or undergo an oral assessment on course topics via video conference before an examination committee during the supplementary examination period (for the committee resit).

The final grade for the course module is awarded according to the calculated value of R_D in accordance with the grading table.

Course Score	*Grade*
100-95	Excellent
94-85	Very Good
84-75	Good
74-65	Satisfactory
64-60	Sufficient
Below 60	Fail
Admission Not Granted	Not Admitted

Working Programme of an Academic Discipline (Syllabus):

Created by Danylo TAVROV, the Head of the Applied Mathematics Department, Assoc. Prof., PhD.

Approved by the Applied Mathematics Department (minutes No 10 dated February 18, 2026).

Endorsed by the Faculty Methodological Commission (minutes № 7 dated February 23, 2026).